OCCLUSION EFFECTS AND THE DISTRIBUTION OF INTERSTELLAR CLOUD SIZES AND MASSES

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ABSTRACT

The frequency distributions of sizes of "clouds" and "clumps" within clouds are significantly flatter for extinction surveys than for CO spectral line surveys, even for comparable size ranges. A possible explanation is the blocking of extinction clouds by larger foreground clouds (occlusion), which should not affect spectral line surveys much because clouds are resolved in velocity space along a given line of sight. We present a simple derivation of the relation between the true and occluded size distributions, assuming clouds are uniformly distributed in space or the distance to a cloud comples is much greater than the size of the complex. Because the occlusion is dominated by the largest clouds, we find that occlusion does not affect the measured size distribution except for sizes comparable to the largest size, implying that occlusion is not responsible for the discrepancy if the range in sizes of the samples is large. However, we find that the range in sizes for many of the published observed samples is actually quite small, which suggests that occlusion does affect the extinction sample and/or that the discrepancy could arise from the different operational definitions and selection effects involved in the two samples. Size and mass spectra from an IRAS survey (Wood et al. 1994) suggest that selection effects play a major role in all the surveys. We conclude that a reliable determination of the "true" size and mass spectra of clouds will require spectral line surveys with very high signal-to-noise and sufficient resolution and sampling to cover a larger range of linear sizes, as well as careful attention to selection effects.

Subject headings: molecular clouds, interstellar medium

1. Introduction

The mass spectrum of density fluctuations, defined in various operational ways as "clouds", is an important function that must be related to the processes by which clouds form and evolve and to the mass spectrum of stars that form within these clouds. A fairly large and growing number of studies of (mostly) molecular clouds yield a differential mass spectrum which, if fit by a power law, has a form $f(m) \sim m^{-\gamma}$, with $\gamma \sim -1.5 \pm 0.2$. These studies are primarily based on masses derived from column densities inferred from ¹³CO spectral line observations and linear size. Some of the results are for surveys that cover a significant area of the Galactic disk, e.g. the second quadrant survey of Casoli et al. (1984). who find $\gamma = -1.4$ to -1.6 in both the Perseus and Orion arms, the ¹²CO first quadrant survey by Sanders et al. (1985) who find $\gamma \sim -1.6$ using virial masses, and the recent comparison of 204 inner- and outer- galaxy molecular clouds by Brand and Wouterloot (1995), who find $\gamma = -1.6$ for outer Galaxy clouds and $\gamma = -1.8$ for all 204 clouds. These surveys together cover a mass range from about 100 M_{\odot} to over 10⁶ M_{\odot} , although each individual study generally covers a much smaller mass range over which a power law is an adequate fit. Other work has concentrated on the mass spectrum of "clumps" within individual clouds complexes, and find similar mass spectra in regions as different in star formation properties as the Maddalena-Thaddeus cloud (Williams, deGeus and Blitz 1994). which shows no evidence for star formation, the ρ -Oph core region (Loren 1989 as revised by Blitz 1993), which is forming low- to intermediate- mass stars, the Rosette Molecular Cloud (Williams et al. 1994, Williams and Blitz 1995), the M17SW cloud (Stutzki and Gusten 1990), and the Orion region (Lada, Bally, and Stark 1991, Tatematsu et al. 1993), all of which are actively forming stars up to large masses, and even lower-mass clumps in MBM 12, a molecular cloud that is not gravitationally bound (Pound 1994). All these studies give $\gamma \sim -1.5 \pm 0.3$ (the flattest being the Williams and Blitz result for the Rosette cloud, with $\gamma \sim -1.3$).

However there is a probable discrepancy when these results are compared with studies of the mass spectra of clouds derived using extinction surveys, which are also based on masses from sizes and column densities. If the distribution of sizes is given by $f(r) \sim r^{\alpha}$ and the cloud internal density n is related to size by $n \propto r^p$, then the mass spectrum is $f(m) \sim m^{\gamma}$ with $\gamma = (\alpha - p - 2)/(3 + p)$. Estimates of p are uncertain and vary from (at least) $p \sim -1.2$ (see Scalo 1985, 1987) to p = 0 (sizes or masses uncorrelated with density, e.g. Casoli et al. 1984, Williams and Blitz 1995), or that the correlation is at least in part an artifact due to selection effects (Scalo 1990). The spectral line studies mentioned earlier give values of α around -2 to -2.5, based either on the published size data when available or on the above transformation between mass and size spectra.

Scalo (1985) presented the frequency distribution of angular surface areas of dark clouds from the catalogues of Lynds (1962) and Khavtassi (1960). The resulting size spectrum, if fit by a power law, has $\alpha \sim -1.4 \pm 0.2$, much flatter than the size distributions inferred from the spectral line surveys. The implied mass spectra ($\gamma \sim -1.2 \pm 0.3$) seem significantly flatter than the spectral line mass spectrum, but, because of the above relationship between size and mass spectra (which gives $\Delta \alpha = (2-3)\Delta \gamma$ for p=-1 to 0), and because size is a directly measured quantity in both types of studies, the discrepancy is more clearly seen in the size spectrum. Feitzinger and Stüwe (1986) studied the statistics of the combined sample of Lynds clouds and their own Southern dark cloud survey, and found a distribution of areas proportional to (area)⁻¹. The corresponding size spectrum has $\alpha = -1$. This gives a mass spectrum index of -1 for any p. Thus the discrepancy with the molecular line survey size or mass spectra is even larger. Other published studies of mass spectra based on extinction are not so clear, but point in the same direction, especially for lower-mass clumps (e.g. Bhatt et al. 1984) for Lynds clouds in Orion, ρ Oph, and Taurus. Drapatz and Zinnecker (1981) give size and mass spectra for several samples based on both extinction and CO.

In the present paper we examine the possibility that this flatter size spectrum seen in extinction is due to the effects of occlusion (smaller clouds being hidden behind large clouds) on the extinction studies; this effect would not affect the spectral line studies nearly as much because in that case two clouds along the same line of sight can be distinguished in velocity space. (Of course occlusion in velocity space can also occur; we discuss this briefly in § 3 below.) We derive an expression for the real size distribution of clouds in terms of the measured distribution that is affected by binary occlusion and derive the range of parameters over which the difference in size spectra between the two approaches can be reconciled.

A relation between the distribution of physical sizes of clouds and their angular sizes is established in § 2, while a relation between the actual distribution of angular sizes and the distribution measured in the presence of occlusion is presented in § 3.

2. "Apparent" sizes of clouds

Consider that $N_1(l)$ is the 'real' size distribution and $N_2(\theta)$ is the "apparent" angular size distribution, without accounting for occlusion. In this seciton we define the relation between $N_1(l)$ and $N_2(\theta)$. This problem is similar to the one discussed in Feitzinger and Stüwe (1986). Due to the geometry of diverging lines of sight, clouds with the same

physical size but at different distances from the observer will fall into different ranges of apparent angular sizes. As a first approximation, assume the "true" properties of clouds to be independent of the distance from the observer. This is probably reasonable for observations in the galactic plane and of nearby individual cloud complexes (e.g. Taurus, Oph, Chameleon, Orion,. . .)

In our model, the distribution of clouds at distance r is given by the product $\varrho(D)N_1(l)$, where $\varrho(D)$ is the total density of clouds at distance D, and we take $N_1(\ell)$ normalized to unity. Then the number of clouds within the distance interval D, D + dD is $\varrho(D)\omega D^2 dD$, where ω is the solid angle. Within this volume, the clouds with sizes from $D\theta$ to $(\theta + d\theta)D$, where θ is the angular size of clouds, will contribute to the apparent angular cloud distribution $N_2(\theta)$. The total number of "projections" with angular sizes $(\theta, \theta + d\theta)$ within the solid angle ω can be found by integrating $\varrho(D)\omega D^2N_1(D\theta)Dd\theta dD$ over the line of sight. Therefore,

$$N_2(\theta)\omega d\theta = \omega \int_{D_{\min}}^{D_{\max}} D^3 \varrho(D) N_1(D\theta) dD d\theta$$
 (1)

A change of variables $D\theta = x$ results in

$$N_2(\theta) = -\frac{1}{\theta^4} \int_{\theta D_{\min}}^{\theta D_{\max}} x^3 \varrho\left(\frac{x}{\theta}\right) N_1(x) dx \tag{2}$$

Assuming $\varrho(x) = \text{constant}$, differentiation gives

$$\frac{1}{D_{\text{max}}} \left(N_2(\theta) \theta^4 \right)' = \theta^3 D_{\text{max}}^3 \varrho N_1(\theta D_{\text{max}}) - \theta^3 D_{\text{min}}^3 \varrho N_1(\theta D_{\text{min}}) \tag{3}$$

For power law $N_1(\ell) = N_1(\ell_{\min}) \left(\frac{\ell}{\ell_{\min}}\right)^{-\gamma}$, the first term is the most important if $\gamma < 3$, whereas if $\gamma > 3$, the second term dominates. The cases of greatest interest here have $\gamma < 3$. Whenever the second term is negligible and $N_1(\ell)$ is a power-law distribution, $N_2(\theta)$ is also power-law with the same index.

Similarly for $D_{\min} = 0$,

$$N_1(\theta D_{\text{max}}) = \frac{1}{\theta^3 D_{\text{max}}^4 \varrho} (N_2(\theta) \theta^4)'$$
(4)

and for any other D_{\min} , the power-law distribution $N_1(l)$ entails a power-law distribution $N_2(\theta)$ with equal slope. Therefore the index of the size distribution is not affected by the differing distances of the clouds in the sample, and the index of the angular size distribution is the same as the index of the linear size distribution. An exception occurs for a delta function linear size distribution, i.e. when all clouds have the same size. In that case the

apparent angular size distribution varies as θ^{-4} (see Bhatt *et al.* 1984). In what follows, we therefore identify $N_2(\theta)$ with the "real" distribution of sizes, with the understanding that clustering of clouds and gradients in the number density of clouds with distance could alter this identification. Obviously, if the distance to a cloud complex is much greater than the extention of the complex, statistics of the "real" size distribution and the "apparent" angular size distribution coinside.

3. Occlusion effect

 $N_2(\theta)$ is the projected apparent angular size distribution of clouds when occlusion is ignored; i.e. it is the angular size distribution corresponding to the "real" linear size distribution. If occlusion is "switched on," some of smaller clouds are hidden behind (or in front) of bigger ones. Let $N_3(\theta)$ be the distribution of projections in the presence of occlusion. Then

$$\pi N_3(\theta)\theta^2 \omega d\theta \tag{5}$$

is the angular area covered by cloud projections with sizes within the range θ , $\theta + d\theta$. The part of the sky not covered by cloud projections with angular sizes greater than θ is

$$1 - \frac{\pi}{A} \int_{\theta}^{\theta_u} N_3(x) x^2 \mathrm{d}x \tag{6}$$

where A is the angular area covered by the survey and θ_u is the upper size limit for the sample. Therefore the number of cloud projections of angular size θ that are *not* occluded by larger clouds is

$$N_2(\theta)\omega d\theta \left(1 - \frac{\pi}{A} \int_{\theta}^{\theta_u} N_3(x) x^2 dx\right)$$
 (7)

Since this is the number of clouds that is seen, equating this to $N_3(\theta)\omega d\theta$ gives

$$N_3(\theta) = N_2(\theta) \left(1 - \frac{\pi}{A} \int_{\theta}^{\theta_u} N_3(x) x^2 dx \right)$$
 (8)

The real size distribution $N_2(\theta)$ can therefore be derived from the apparent (occluded size distribution $N_3(\theta)$ from

$$N_2(\theta) = \frac{N_3(\theta)}{1 - \frac{\pi}{A} \int_{\theta^u}^{\theta_u} N_3(x) x^2 dx}$$

$$\tag{9}$$

The second term in the denominator is just the fraction of the survey area A covered by clouds with sizes greater that θ . The largest value this fraction can have occurs at $\theta = \theta_{\ell}$,

the minimum size detected in the survey, for which the second term is the total area filling factor of clouds detected in the survey (< 1).

It is also possible to derive the observed distribution $N_3(\theta)$ that would result from a given real distribution $N_2(\theta)$, as shown in the Appendix. However, that formulation is not as useful for the purposes of the present paper because the solution involves the unknown properties of the real distribution.

To illustrate the properties of the $N_2 - N_3$ relation, assume that the observed occluded distribution is a power law, $N_3(\theta) = c_3 \theta^{-\gamma_3}$. Then

$$N_2(\theta) = \frac{c_3 \theta^{-\gamma_3}}{1 - \frac{\pi}{A} \frac{c_3}{(3 - \gamma_3)} (\theta_u^{-r_3 + 3} - \theta^{-\gamma_3 + 3})}$$
(10)

The total areal filling fraction is

$$f_{3,tot} = \frac{1}{A} \int_{\theta_{\ell}}^{\theta_{u}} N_{3}(\theta) \pi \theta^{2} d\theta = \frac{\pi c_{3}}{A(3 - \gamma_{3})} \left(\theta_{u}^{-\gamma_{3} + 3} - \theta_{\ell}^{-\gamma_{3} + 3} \right). \tag{11}$$

The second term is negligible for $\theta_{\ell} \ll \theta_u$ and $\gamma_3 < 3$. So, from eqs. (10) and (11), we see that for θ significantly smaller than θ_u , $N_2(\theta) = N_3(\theta)/(1 - f_{3,tot})$; i.e. for small clouds the real number of clouds is larger than the observed number by a factor $(1 - f_{3,tot})$, but the power law index is unaffected. The probability of a small cloud to be hidden by a large cloud is independent of its size if its size is much smaller than θ_u because the areal filling is dominated by the largest clouds (if $\gamma_3 < 3$).

To see this more clearly, consider the local logarithmic slope of the real distribution at size θ (i.e. the exponent of a local power law fit at that size). From eq. (10) we obtain

$$\gamma_{2}(\theta) = \frac{d\ell n N_{2}(\theta)}{d\ell n \theta} = \gamma_{3} + \frac{\pi c_{3}}{A} \frac{\theta^{-\gamma_{3}+3}}{\left[1 - \frac{\pi c_{3}}{A(3-\gamma_{3})} \left(\theta_{u}^{-\gamma_{3}+3} - \theta^{-\gamma_{3}+3}\right)\right]}$$

$$\equiv \gamma_{3} + \Delta \gamma(\theta)$$
(12)

The maximum value of the change in exponent $\Delta\gamma(\theta)$ occurs for θ near θ_u , at which size $\Delta\gamma(\theta) = \pi c_3 \theta_u^{-\gamma_3+3}/A \approx (3-\gamma_3) f_{3,tot}$ (for $\theta_\ell \ll \theta_u$ and $\gamma_3 < 3$). If $f_{3,tot} \approx 0.5$, as is typical for dark cloud surveys (not selected according to opacity class or size), then the dark cloud power law $\gamma_3 \sim 1.4$ gives $\Delta\gamma \approx 1.6 f_{3,tot} \sim 0.8$. While this is about the value needed to reconcile the extinction size distribution with the spectral line size distribution, it only occurs very close to θ_u . At smaller θ , say $x\theta_u(x < 1)$, $\Delta\gamma$ is reduced by a factor of $x^{-\gamma_3+3} \sim x^{1.6}$ for the parameters chosen. So even for clouds half or a third of the size of the largest clouds, $\Delta\gamma$ is too small to account for the discrepancy, and for x = 0.1, $\Delta\gamma$ is essentially negligible.

The same considerations hold even if the observed distribution $N_3(\theta)$ is not a power law, as long as it is not locally too steep ($\gamma > 3$): the real size distribution tracks the observed distribution (although at larger amplitude) except for sizes close to θ_u , at which sizes the real distribution is steeper than the observed distribution.

We would be tempted to conclude that occlusion cannot account for the discrepancy, except for the fact that the range in sizes in the observed surveys is actually quite small. For both types of surveys, the cloud masses are proportional to the square of some characteristic size times a column density, so the range in sizes, which is a directly observed datum, only corresponds to the square root of a given range in the masses (which is what is usually displayed). Since, for the published spectral line surveys of clumps within cloud complexes, power laws are only good fits over a limited mass range (limited by small numbers at the largest masses and resolution incompleteness and other effects at small masses), usually a factor of 10–100, the range in sizes is not very large. The range in sizes for a few early surveys is listed in Drapatz, and Zinnecker (1984). The range in sizes for the line surveys of Stutzki and Gusten (1990), Lada et al. (1991), Tatematsu et al. (1993), Williams et al. (1984) and Williams and Blitz (1995) is less than a factor of 10, although the range in mass used to derive the mass spectra is larger in some of the surveys. This suggests that the mass distributions derived from spectral line surveys will be very sensitive to the definition of, and systematic uncertainties in the measurement of, cloud sizes. For the extinction sample the range of sizes over which the power law size spectrum is applicable is less than a factor of about 10 in all cases, even for the full sample of the Lynds and Khavtassi surveys, and various selection effects come into play at smaller and larger masses (see Scalo 1985, § III.B.2. for a discussion).

Thus we conclude that the discrepancy between size distributions derived from extinction surveys and spectral line surveys may be due to occlusion effects in the extinction surveys because the minimum size is not much smaller than the maximum size in both types of surveys, or because of different operational definitions of size in the two types of surveys. Actually these two possibilities are not independent because the size range is related to how clouds are defined. It is worth pointing out that in some of the spectral line surveys the noise level is so large that the surveys are really only observing the "tips of the mountain range" if the column density map is thought of as a 2-dimensional surface with height equal to column density. For example, in the Rosette data (Blitz and Stark 1986), the rms noise is only about a factor of 2 smaller than the average peak line temperature, so the cloud sizes may be severely affected. For the dark clouds, identification of the cloud boundary is usually much less affected by "noise" (in this case fluctuation in star densities), except for the lowest-opacity clouds. Thus even though the line surveys have the advantage of separating clouds in velocity space it is not clear that they give more realistic size and

mass spectra compared to extinction surveys.

Evidence that the empirical cloud mass spectra are sensitive to selection effects comes from the following two examples.

Clemens and Barvainis (1988) compiled a catalogue of isolated small dark cloud ("globules") identified on POSS plates and compiled properties based on their CO observations. For clouds with mean size larger than 3.5 arcmin (smaller size clouds are probably affected by incompleteness), we can fit the frequency distribution of angular sizes, and hence linear sizes if the clouds are uniformly distributed in distance, by $f(r) \sim r^{-2}$, which gives a power law mass spectrum with $\gamma = -1.5$ to -1.7 for p = -1 or 0. These clouds were selected to be small and isolated, so occlusion should not be important. Since this result agrees with the molecular line surveys, it suggests that the flatter size and mass spectra derived from general extinction surveys are products of occlusion effects, if selection effects are unimportant in the estimation of properties from CO.

However the survey of 255 IRAS cloud cores by Wood, Myers, and Daugherty (1994), which derives sizes and masses based on IRAS 100 μ m optical depth for clouds with $A_V \gtrsim 4$ mag, yields a frequency distribution of areas $f(A) \sim A^{-0.54}$, or $f(r) \sim r^{-0.08}$, which is extremely flat compared to not only the molecular line surveys, but even extinction surveys. For constant column density, as Wood et al. find, p=-1, so $f(m) \sim m^{-0.54}$, consistent with their directly determined (from individual areas and column densities) $f(m) \sim m^{-0.49}$. The sizes and masses for the fits have ranges of well over 1000. Since all the cores are optically thin at 100 μ m, occlusion cannot be a factor; a small core behind a larger core would be seen as a column density enhancement of about a factor of two, because all the cores in the sample apparently have about the same column density. This result suggests that all the surveys, whether based on extinction, molecular line, or IRAS, are affected by selection effects.

4. Velocity occlusion

The same argument used above for purely spatial occlusion can be somewhat extended to include the effects of blending in velocity space for spectral line surveys. In this illustrative example we assume that each identified "cloud" or "clump" (for convenience we use the latter term in what follows) has an internal velocity dispersion $\Delta v(\theta)$ which is strictly correlated with the size of the clump, as found in several surveys, at least for clumps in which self-gravity is important. In that case the fraction of the total survey volume of

the data cube AV (A = area of the survey in the plane of the sky, V = radial velocity extent of the survey) occupied by occluded clumps of size θ is

$$f_v(\theta) = \frac{1}{AV} \int_{\theta}^{\theta_u} \pi \theta^2 N_v(\theta) \Delta v(\theta) d\theta$$

where $N_v(\theta)$ is the size distribution found in the (blended) survey. The real size distribution is then

 $N_2(\theta) = \frac{N_v(\theta)}{1 - f_v(\theta)}$

The maximum value of $f_v(\theta)$ occurs at θ_ℓ and is the total volume filling factor of observed clumps in the data cube. Since this number is small for the surveys we are aware of (see Fig. 7 in Williams and Blitz 1995), the effect of this type of occlusion (due to finite internal velocity dispersion of the clumps) on the derived size distribution must be negligible, at least for velocity resolutions much smaller than the minimum Δv . However, this analysis does not account for the fact that clumps with similar centroid velocities may lie along the same line of sight. Taking this effect which probably dominates the blending in velocity space, into account would involve calculting the probability that, for a prescribed centroid velocity distribution, two clouds along a given line of sight have a centroid velocity difference smaller than the sum of the linewidths of the two clouds (which is a function of θ), a calculation which we postpone to a later publication.

5. Conclusions

Our study has examined the effect of occlusion on extinction surveys. The predicted change in the shape of the frequency distribution of cloud sizes for extinction surveys compared to spectral line surveys is small, except very near the maximum cloud size. Thus the discrepancy between the empirical results for the two types of surveys probably cannot be attributed to occlusion in the extinction survey, if the size range of both types of survey is large. However an examination of the literature shows that many of the observed surveys employ a very limited range of sizes. In these cases the discrepancy might still be due to occlusion. On the other hand, some of the spectral line surveys do include clouds with a fairly large range of sizes (e.g. Brand and Wouterloot 1995), and these surveys do find size and mass spectra much steeper than the dark cloud results. Furthermore, the IRAS cloud-core survey of Wood et al. (1995) gives size and mass spectra which are much flatter than both the extinciton and line survey results. This suggests that the inferred shapes of the size and mass spectra of clouds are affected by the manner in which clouds are defined and by the selection and noise effects inherent in both types of surveys.

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APPENDIX

Rather than solve for the real distribution function in terms of the observed (occluded) distribution) it is possible to derive the observed distribution $N_3(\theta)$ that would result from a given real distribution $N_2(\theta)$. Differentiating eq. 9 with respect to θ gives

$$N_3'(\theta) = N_3(\theta) \left[\frac{\pi \theta^2}{A} + \frac{N_2'(\theta)}{N_2^2(\theta)} \right] N_2(\theta)$$
 (1)

Integrating this equation, with a lower integration limit θ_{ℓ} , gives

$$\frac{N_3(\theta)}{N_3(\theta_\ell)} = \frac{N_2(\theta)}{N_2(\theta_\ell)} \exp\left\{\frac{\pi}{A} \int_{\theta_\ell}^{\theta} N_2(x) x^2 dx\right\}$$
 (2)

We can obtain $N_3(\theta_\ell)/N_2(\theta_\ell)$ by imposing the condition that the largest cloud in the sample cannot suffer any occlusion, i.e. by substituting $N_3(\theta_u) = N_2(\theta_u)$ at $\theta = \theta_u$ in eq. A2. This condition results in

$$\frac{N_3(\theta_\ell)}{N_2(\theta_\ell)} = exp\left\{\frac{\pi}{A} \int_{\theta_\ell}^{\theta_u} N_2(x) x^2 dx\right\} = exp(-A_{tot}/A),\tag{3}$$

where A_{tot} is now the total area covered by all clouds in the unoccluded (real) distribution, and may be greater than the survey area A. Dividing the integral from θ_{ℓ} to θ_{u} into parts from θ_{ℓ} to θ and from θ to θ_{u} and substituting into eq. A2 gives

$$N_3(\theta) = N_2(\theta) \exp\left\{-\frac{\pi}{A} \int_{\theta}^{\theta_u} N_2(x) x^2 dx\right\} = N_2(\theta) \exp[-A(>\theta)/A], \tag{4}$$

where $A(>\theta)$ is the area covered by clouds with sizes greater than θ in the unoccluded distribution and may xxx by greater than A. For a power law $N_2(\theta) = c_2 \theta^{-\gamma_2}$ we find

$$N_3(\theta) = N_2(\theta) \exp\left\{\frac{\pi c_2}{(3 - \gamma_2)A} \left(\theta_u^{-\gamma_2 + 3} - \theta^{-\gamma_2 + 3}\right)\right\}.$$
 (5)

The local logarithmic slope of the predicted occluded distribution is then (assuming $\theta_{\ell} \ll \theta_{u}$ and $\gamma_{2} < 3$)

$$\gamma_3(\theta) = \frac{d\ell n N_3(\theta)}{d\ell n \theta} = \gamma_2 - \frac{A_{tot}}{A} \left(\frac{\theta}{\theta_u}\right)^{-\gamma_2 + 3}.$$
 (6)

Once again we see that although the change in local logarithm slope may be large near θ_u , the effect becomes increasingly negligible for $\theta \ll \theta_u$.

However this formulation is not as useful as that given in the main text (which expressed $N_2(\theta)$ in terms of $N_3(\theta)$) because the total covering fraction of the real distribution is unknown, although it can be evaluated for a model which specifies the total number of clouds in the distribution (again unknown from observations).

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